Written by Yarden Nativ, mail: nativyardenac@gmail.com

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## Frequency Domain III: Spectral Analysis

Special thanks: many of the following notes are based on Tal Dalal presentation for first degree SDA course.

## Uses of spectral analysis in neuroscience studies

Spectral analysis refers to a group of methods which uses the frequency values of a signal to characterize it. The properties of a signal in the frequency domain allow to indicate changes in its periodic behavior, to correlate between the signal and other signals by its frequencies' powers or phases and so on.

For example, a hot topic in fMRI studies today is regarding functional connectivity, the way different regions are operating simultaneously to allow some complex behavior such as attention. Commonly the most common use of fMRI connectivity is based on Pearson correlation coefficients between the bold signals of two regions. Yet, Muller et al. 2001 offer the use of spectral analysis for such a computation. The main advantage of such a method is that the Bold signal in spectral analysis is "whitened" before the correlation computation by switching to the spectral domain and using a spectral density estimation. The method offers to use spectral analysis instead of Multivariate analysis for better identification of local functional networks.

In the paper they use coherence coefficients for certain frequencies suggested by the experimental design, and by which they created a cluster of voxels with high coherence coefficient- meaning voxels which their bold signal has a very similar frequency to the one of the stimuli presented. For this cluster they compute the phase lead of the suggested frequencies. The harmonic is then defined by the division of the phase with the lead frequency with maximal coherence.

The idea of localization of small network is demonstrated with the following visual stimulus (a matrix containing 7X12 red L-shaped angles on black background rotating randomly every 100 ms):

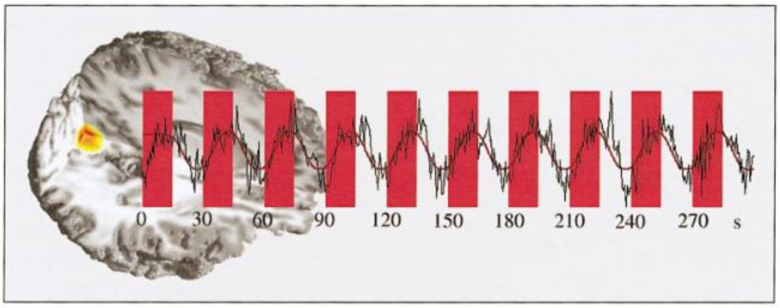



FIG. 2. Fitted response and time course for the most activated voxel. The right visual field was stimulated every 30 s for a duration of 15 s.

Those are the relations between the phases of the cluster voxels with top coherence to the stimulus (corresponding to V1, V2 and lateral occipital sulcus):

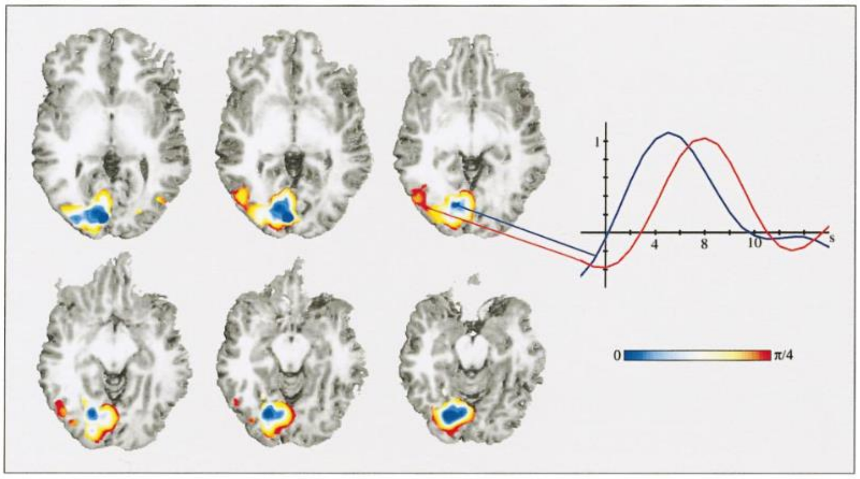



FIG. 6. The phase lead can be interpreted as the time displacement of the hemodynamic response function. This temporal shift is already to be seen in the trial average.

## The change of oscillations in time (PSD and spectrogram)

Instead of using the Fourier transform, we can use an equivalent measurement of PSD- power spectral density. This will indicate what are the main frequencies of the signal. The PSD is based on Parseval's theorem which allows to compute the power of frequencies straight from the time domain:

$$\text{Continuous: } \int_{-\infty}^{\infty}|x(t)|^2 dt = \int_{-\infty}^{\infty}|X(f)|^2 df$$

$$\text{Discrete: } \sum_{n=0}^{N-1}|x[n]|^2 = \sum_{n=0}^{N-1}|X[k]|^2$$

So why do we need density? Because according to Parseval's theorem, if we take two signals with the same frequency, but one is two times longer than the other- than the magnitude will be 4 times higher. For this, we prefer to use power instead of energy while:

$$power = \frac{energy}{time}$$

Using the Wiener-Khinchin theorem, we now can calculate the power spectrum as the Fourier transform of the auto-correlation function:

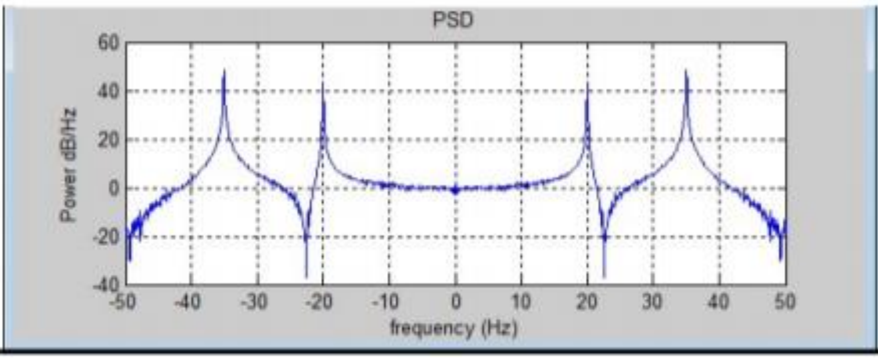
$$S(f) = \int_{-\infty}^{\infty} R(\tau)e^{-2\pi i f\tau}d\tau = \mathcal{F}\big(R(\tau)\big)$$

Nevertheless, the sampling frequency influences the result of S(f) so we normalize once again for each frequency:

$$P_{x,x}(f) = \frac{S_{x,x}(f)}{f_s}$$

PSD (also known as Periodogram) computational example:

```
Fs = 100;        t = 0:1/Fs:10;
x = 100*cos(2*pi* 20*t)+ 200*cos(2*pi* 35*t)+ randn(size(t));
Pxx = abs(fftshift(fft(x))).^2/(length(x)*Fs);
freq = -Fs/2 : Fs/(length(Pxx)-1) : Fs/2;
plot(freq, 10*log10(Pxx));
xlabel('frequency (Hz)'); ylabel('Power dB/Hz'); title('PSD');
```



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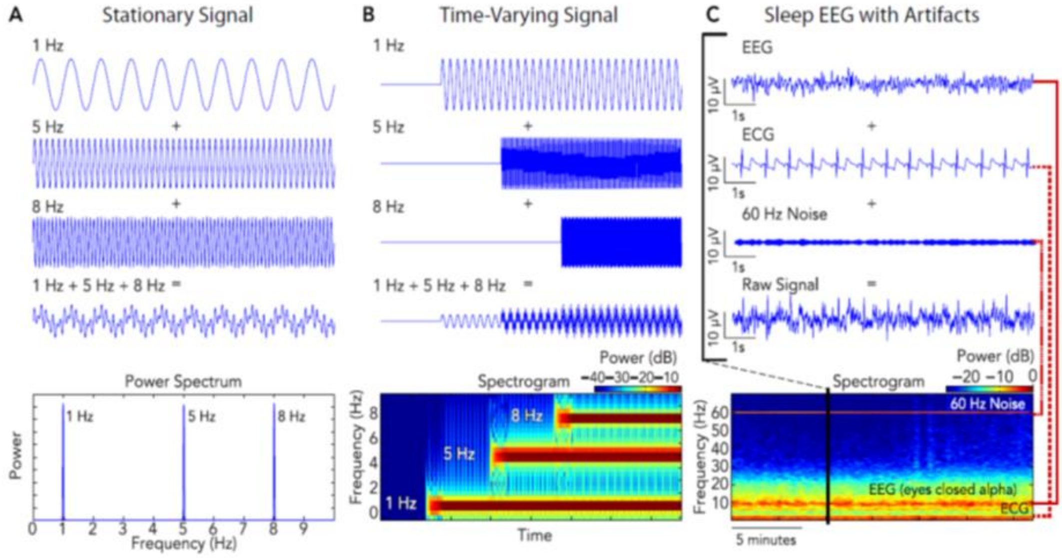
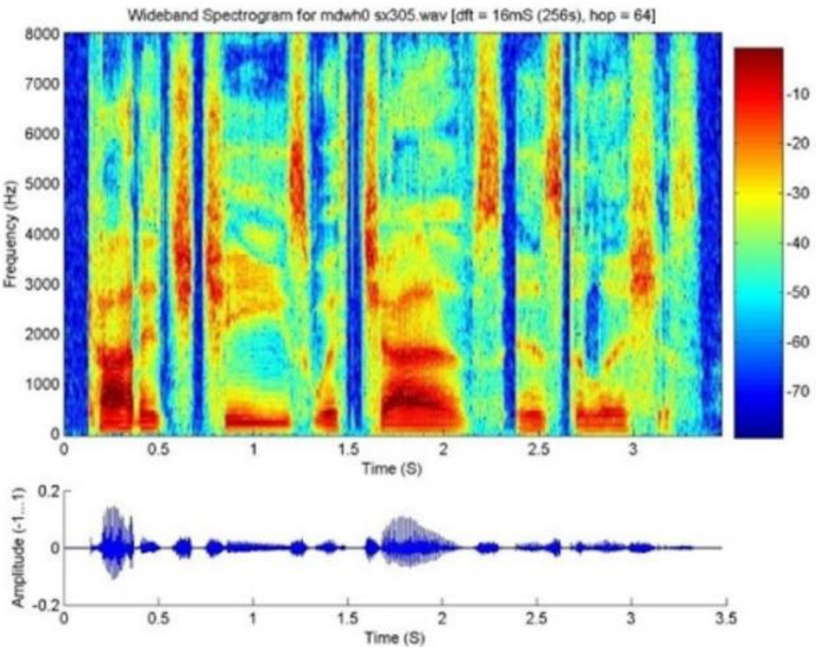
What a spectrogram is? If the signal is time varying, we can show the changes in the frequencies in time. Therefore, we use PSD when we assume stationary signal and spectrogram for a time varying signal:




# Spectrogram of speech



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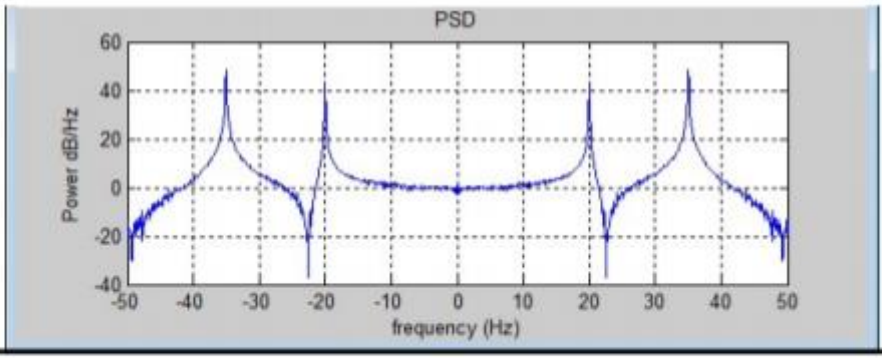
## The STFT problem (Welch, Bartlett and Multitaper)

**Class Discussion: Look at the PSD computational example again, why the frequencies other than 20 and 35 are note almost zero?**

```
Fs = 100;        t = 0:1/Fs:10;
x = 100*cos(2*pi* 20*t)+ 200*cos(2*pi* 35*t)+ randn(size(t));
Pxx = abs(fftshift(fft(x))).^2/(length(x)*Fs);
freq = -Fs/2 : Fs/(length(Pxx)-1) : Fs/2;
plot(freq, 10*log10(Pxx));
xlabel('frequency (Hz)'); ylabel('Power dB/Hz'); title('PSD');
```




When we compute the PSD, practically we compute a FFT for a finite sampling of the wave, so that we use a rectangle window for Fourier transform computation. As we studied in filters, we create a rectangle filter to the wave, so we add a sinc function to the original signal.

For this reason, we use three different methods for computing the PSD:

- Bartlett: Using non overlapping windows to compute the STFT for each one and average between them.
- Welch: Using a sliding window we compute the STFT for each window and we average between the windows.
- Multitaper: Filtering the data by convolution into bandpass filters (Slepian sequences), calculating the periodogram for each one and averaging between them (recommended explanation: https://raphaelvallat.com/bandpower.html)

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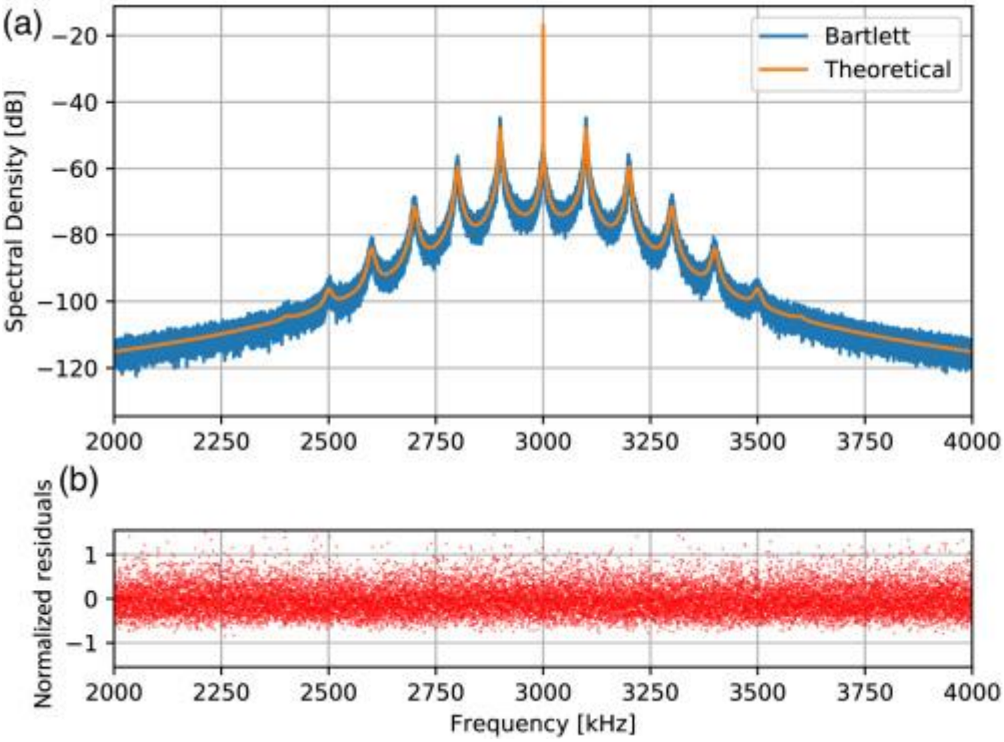
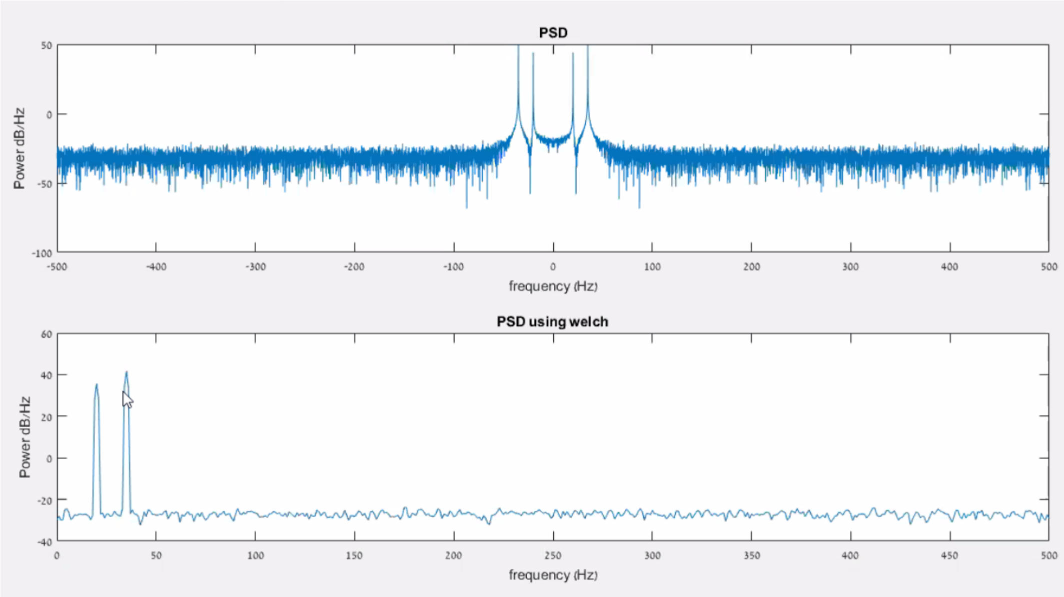
Example for Bartlett:




Example for Welch:



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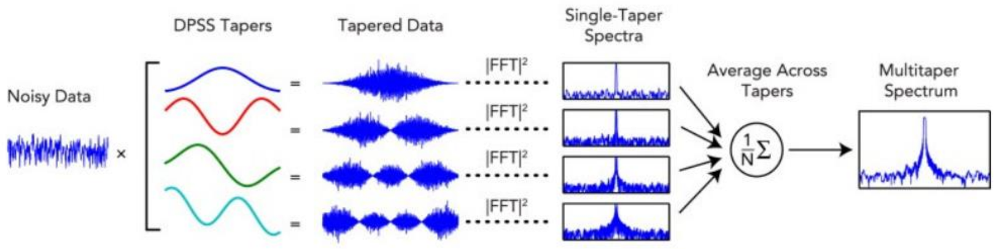
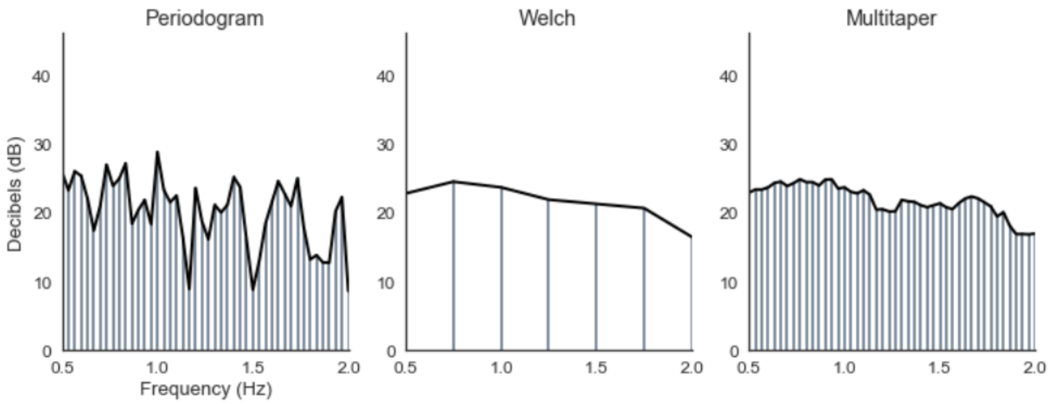
Example for multitaper:



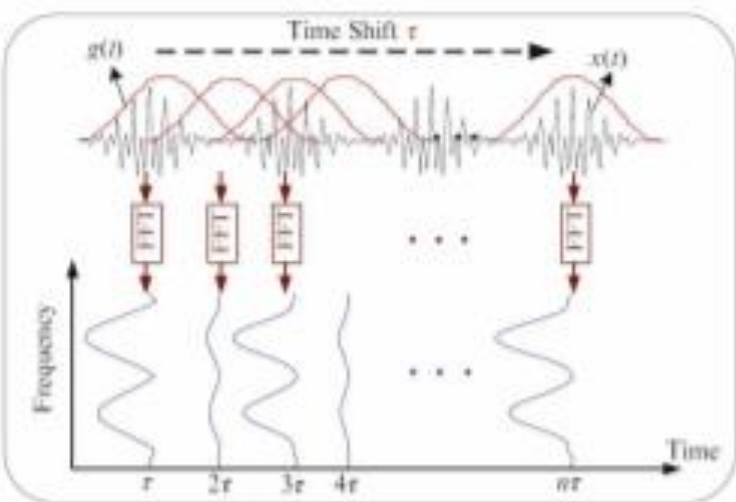



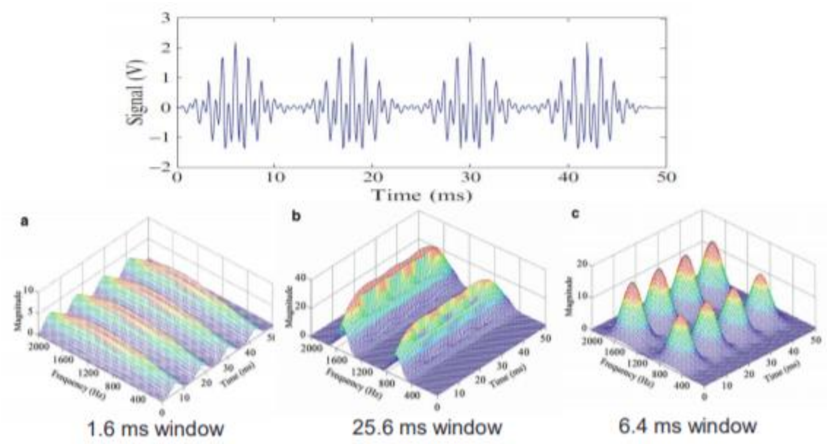

## The STFT trade-off

### Class Discussion: How do we decide what method to use?

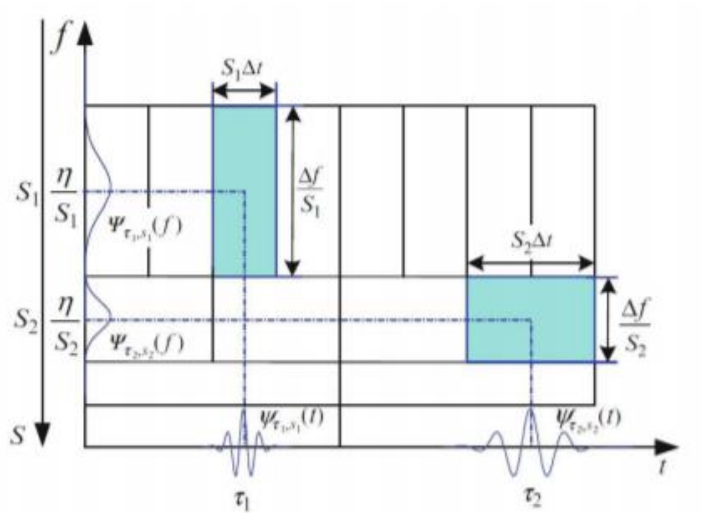
The main difference between the Bartlett window and the Welch is the statistical analysis, because in the Bartlett window there is no overlap between the windows. Therefore, the Welch gives better SNR, but it is less useful.

What is the difference between the windows and the Multitaper? Due to STFT we have trade-off between the frequency resolution and the time resolution. For example in the following spectrogram, the longer the windows are the better the spectral resolution is:






The multitaper method uses an idea like wavelets which allows to avoid this trade-off by filtering to a set of known wavelets. For example, the following wavelet presents the change in resolutions along the time:




## Class Exercise:

If you use the Welch or Bartlett window, what window size will you pick for a signal with sampling rate Fs to achieve the following spectral resolution?

a. 1 Hz using Bartlett window
b. 0.5 Hz using Welch window with overlap of 50%
c. 2 Hz using Welch window with overlap of 50%

**Solution:**

a. The same of Fs
b. Twice as Fs
c. Half of Fs

## Class Discussion: If we have non-stationary noise in the data what method for PSD calculation is better? And when there is a stationary noise?

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## Class Exercise:

**2005 exam:**

The EEG of the Golden Unicorn is made up of three frequencies 7, 18 & 100Hz. The amplitude of each component (frequency) is 10µV. The waveform is sampled at 150 samples/sec for 1 second. Sketch and explain the power spectrum of the EEG. What are the X & Y axes? What is the X scale resolution? What is the maximal frequency?

**Solution:**

Due to the sampling rate, we have aliasing: The 100 Hz frequency will be at 50 Hz. The X and Y axes are the frequency and power in DB/Hz, and the X scale resolution is 1 Hz because the division between 150 and 150 is 1. The maximal frequency is as calculated, 50 Hz.

## The correlation of oscillations between two signals (Cross spectrum and Coherence)

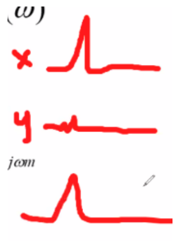
The cross spectrum is a method which allows to examine a causal relation between two frequencies by multiplying the Fourier transforms of the two signals. The cross spectrum indicates how much can one signal predicts the other one with the same frequency:

$$\hat{S}_{y,x}(\omega) = \frac{1}{N} Y(\omega) X^*(\omega)$$

This cross spectrum is in fact the FFT of the cross-correlation:

$$S_{x,y}(\omega) = \sum_{-\infty}^{\infty} R_{x,y}(m) \cdot e^{-j\omega m}$$

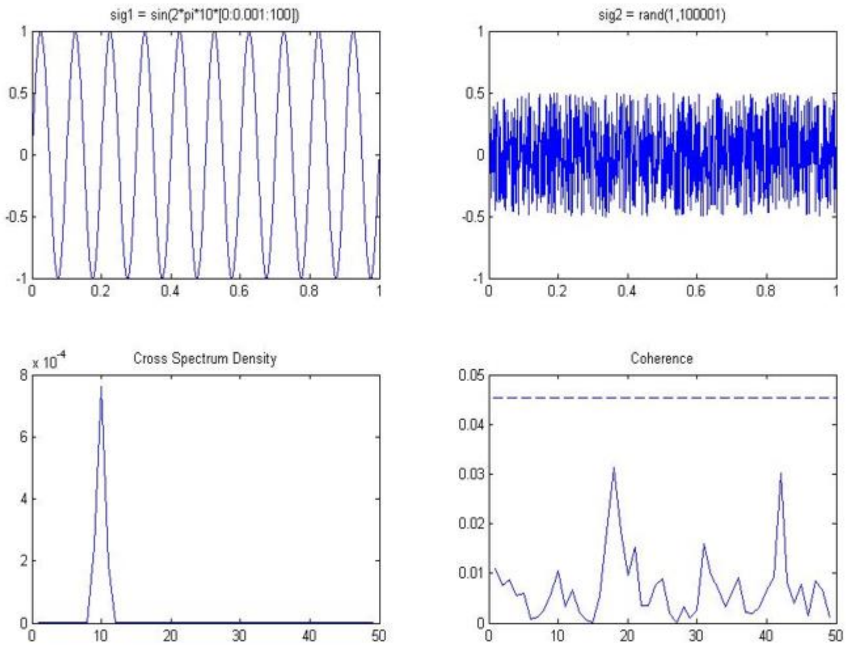
Nevertheless, cross spectrum is less useful due to the high dependency of the magnitude of each signal. For example, in the following two signals, the result will be highly influenced by the amplitudes of the signals:



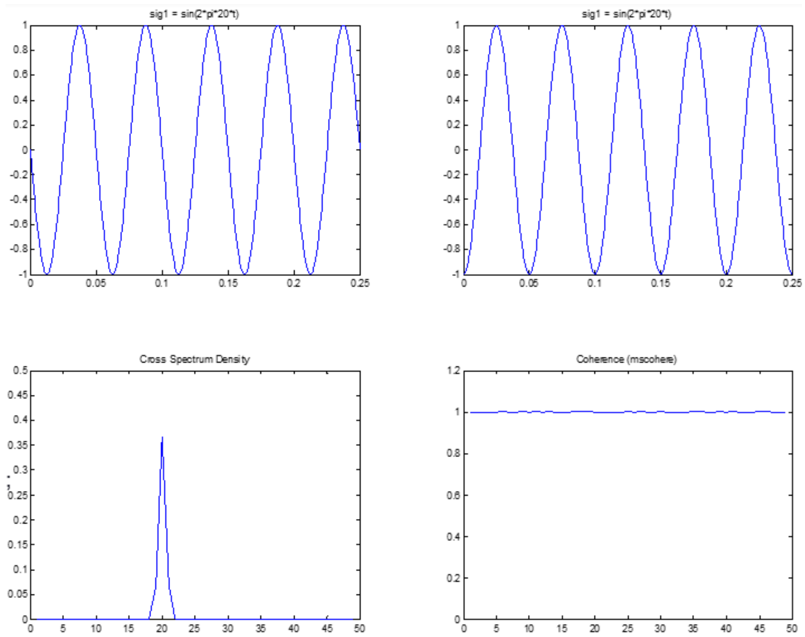
Therefore, more commonly we use the coherence, which normalizes the cross spectrum to the spectral density of the signals:

$$C_{xy} = \frac{|S_{xy}|^2}{S_{xx}S_{yy}} \qquad 0 \le C_{xy} \le 1$$

For example, if we have white noise and a sine signal, we can see in the cross-spectrum peak in 10 Hz, the frequency of the sine. Yet, the coherence presents no synchrony between the two signals:




If we have two signals with phase difference, and the two signals are pure sine signals, we can see full coherence for all the frequencies:



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## Class Exercise:

**2007 exam:**

Given the two signals $a(t) = Asin(20\pi t + \alpha)$ and $b(t) = Bsin(20\pi t + \beta)$, the coherence between the two signals at 10Hz is:

a. The amplitude depends on A & B and the phase depends on α & β.

b. The amplitude does not depend on A & B and the phase depends on α & β.

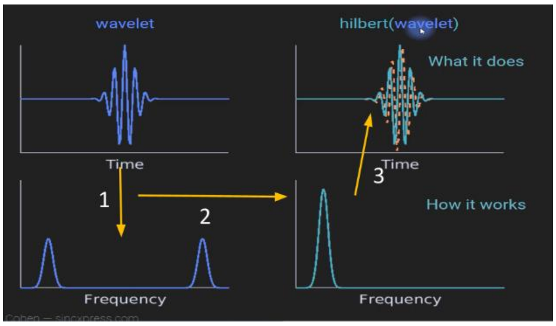
c. The amplitude depends on A & B and the phase does not depend on α & β.

d. The amplitude does not depend on A & B and the phase does not depend on α & β.

e. None of the above.

## Addressing the phase (Hilbert transform and phase locking)

The Hilbert transform allows to transform a real time signal into a complex time signal given a reference point on the signal which represents 0 phase. We use the Hilbert transform for data which is band filtered and that has power, because for broadband signals the Hilbert transform is useless. The Hilbert transform performs a FFT to the original signal, removes the negative frequencies, doubles the positive and inverse the transform to get the complex signal. The phase at each time will be of the frequency with the highest power at that time point:




Now we can use the Hilbert transform to calculate phase locking by quantifying the variance of the phases for a given spike. Perfect synchronization of spikes to the oscillations will be a delta function, and random synchronization will be a uniform distribution: